International Journal of Applied Mathematics
& Statistical Sciences (IJAMSS)
ISSN(P): 2319-3972; ISSN(E): 2319-3980
Vol. 2, Issue 5, Nov 2013, 117-124

International Academy of Science, Engineering and Technology
Connecting Researchers; Nurturing Innovations

AN UNSTEADY BOUNDARY LAYER FLOW OF AN INCOMPRESSIBLE MICROPOLAR FLUID NEAR STAGNATION POINT WITH ELECTROMAGNETIC FIELDS

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ABSTRACT

An unsteady boundary layer flow of an incompressible micropolar fluid near the forward stagnation point under electromagnetic field has been studied. The velocity of the flow is assumed to have started impulsively from rest and is maintained thereafter. Using non-similarity transformations, the governing boundary layer flow equations are reduced to boundary value problem (BVP). This system involves time and space variables. The missing initial conditions are obtained using Newton's method to satisfy the end conditions of the boundary. The results are compared with available results to confirm the validity of the numerical code developed and the approach used. Velocity profiles, micro-rotation profiles and skin-friction coefficient for various values of parameters involved are presented. Smooth transition from unsteady to steady flow for large time solutions is observed. The effect of electromagnetic parameter on the flow field is also presented and is observed that with the increase of this parameter, flow reduces near the wall.

KEYWORDS: Unsteady Boundary Layer Flow, Micropolar Fluid, Electro Magnetic Fields

INTRODUCTION

The quality of a product or efficiency of the production in extrusion process depends on fluid flow and mass transfer of the fluid involved. The fluid contains suspended particles, dust, metal particles etc..., such a fluid is called micropolar fluid. This is an important issue to be discussed in extrusion process. The theory takes into account the microscopic effects arising from the local structure and micro-motions of fluid particles of non-Newtonian flows. This provides basis for an efficient modeling and performance of products/instruments in the fields of exotic lubricants, polymers, liquid crystals and colloidal suspension solutions.

There is a wide discrepancy between theoretically predicted and experimentally observed results. This may be because of unrealistic assumptions made in the modeling of the problem and approximations used in solving the resultant mathematical systems. Though the theory of micropolar fluids was introduced by Eringen[1], it has become popular research topic in recent times due to its wide range of applications in production activities. A comprehensive study of micropolar, fluids has been presented by Guram and Smith [2] and have obtained numerical solutions of stagnation Point flows of micropolar fluids under strong and weak interactions. The solutions were obtained using fourth order Runge-Kutta method. Gorla [3] has obtained numerical solutions for micropolar boundary layer flow at a stagnation point on a moving wall. Recently, Nazar and Amin et al. [4] have studied an unsteady boundary layer flow over a stretching sheet in micropolar fluid and obtained numerical solutions using Keller Box method. Many problems of practical interest are unsteady. In fact, there is no natural or practical applications, which do not involve unsteadiness. Steady and unsteady Newtonian flows are studied well by Katagiri[5], whereas unsteady flows of micropolar fluids are attempted by very few.

The present paper studies the effect of electromagnetic fields on unsteady boundary layer flow of micropolar fluid near the forward stagnation point of a wall. The velocity of the flow is assumed to have started impulsively at the stagnation point from rest and maintained steady state thereafter. The governing equations of the model are non-linear coupled partial differential equations(PDE). It is very difficult to solve the resultant nonlinear coupled system as it is, however, one can consider briefly the status of a “model” which can be judged as an adequate starting point for engineering calculations as referred by Blackmann et al in [6]. The importance of minimizing discrepancy between theoretically predicted and experimentally observed results and inclusion of electromagnetic field effects are explained by Naidu et al [7]. Imposing certain assumptions and using appropriate transformations the resultant system is reduced to standard form. This system is solved numerically for various values of material and electromagnetic field parameters. The profiles for velocity, micro-rotation and skin friction coefficient are plotted and presented in graphical form.

Governing Equations: Consider a 2-Dimensional unsteady boundary layer flow of a micropolar fluid near stagnation point as shown in the figure.1. The flow is assumed to have started impulsively at time t=0 from rest along the wall x-direction and perpendicular to it y-direction. Using boundary layer theory and approximations, the basic equations governing the fluid flow are reduced to the following form as in Lok Yian et.al. [8].

Physical Model

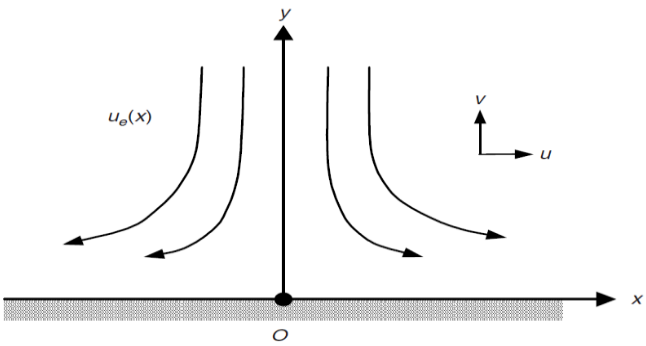

Figure 1: Physical Model and Coordinate System

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=u_e\frac{du_e}{dx}+\left(\frac{\mu+k}{\rho}\right)\frac{\partial^2 u}{\partial y^2}+\frac{k}{\rho}\frac{\partial N}{\partial y}-\frac{\sigma B^2{}_0}{\rho}u \tag{2}$$

$$\rho j\left(\frac{\partial N}{\partial t}+u\frac{\partial N}{\partial x}+v\frac{\partial N}{\partial y}\right)=\gamma\frac{\partial^2 N}{\partial y^2}-k\left(2N+\frac{\partial u}{\partial y}\right), \tag{3}$$

Subject to the boundary conditions

for $t \le 0: u(x,\ y,\ t)=v(x,\ y,\ t)=N(x,y,t)=0,\ \forall$ x, y and

$$t>0:\ u_w=0,\ v=0,\ \ N=-n\frac{\partial u}{\partial y}\ \ at\, y=0 \tag{4}$$

$u \to u_e(x),\ N \to 0,$ as $y \to \infty$

Where u and v are velocity components along x and y –directions, t: time variation. N: component of micro-rotation vector normal to the xy-plane, ρ : density, μ, ν and k : absolute, kinematic viscosity and vortex viscosity respectively and μ = ρ ν are constants, n: a constant 0 ≤ n ≤ 1, and j: micro-inertia density. If n = 0 called strong concentration as in Gurham and Smith[2]. For which N = 0 near the wall implies that the concentrated flows of microelements close to the wall surface unable to rotate as in Lok Yian et al [8]. For the case n = ½ indicates vanishing anti-symmetrical part of the stress tensor and it denotes weak concentration of microelements. We shall consider for the case n = 0 and n = ½ in which it is assumed that the physical quantities ρ, μ, k, γ .

Introducing the following dimensionless variables

$$\psi = (cv\xi)^{1/2} xf(\xi,\eta),\ N = (c/\xi v)^{1/2} cxg(\xi,\eta), \eta = (c/v\xi)^{1/2} y,\quad \xi = 1 - e^{-\tau},\ \tau = ct, \tag{5}$$

such that c (> 0) a positive constant, the stream function (Ψ) is defined such that

u = $\dfrac{\partial\psi}{\partial y}$ *and* $v = -\dfrac{\partial\psi}{\partial x}$ automatically satisfies (1) and the above system of equations (2)-(3) get reduced to

$$(1+K) f''' + (1-\xi)\frac{\eta}{2} f'' + \xi(ff'' - f'^2) + Kg' - \xi(1-\xi)\frac{\partial f}{\partial \xi} - M\xi f' = 0 \tag{6}$$

$$\left(1+\frac{K}{2}\right)g'' + (1-\xi)\left(\frac{g}{2}+\frac{\eta}{2}g'\right) + \xi(fg' - f'g) - K\xi(2g + f'') = \xi(1-\xi)\frac{\partial g}{\partial \xi} \tag{7}$$

with the boundary conditions

$$f(\xi,0)=0,\ f'(\xi,0)=0,\ g(\xi,0)=-nf''(\xi,0),\quad f'(\xi,\infty)=1,\ and\ g(\xi,\infty)=0. \tag{8}$$

Where electro-magnetic and material parameters are: $M = \dfrac{\sigma B}{\rho c},\ K = \dfrac{k}{\mu}$.

The physical quantity skin friction coefficient is defined as

$$C_f = \frac{\tau_w}{\rho u_w^{\ 2}/2}, \text{such that } \tau_w = \left[(\mu+k)\frac{\partial u}{\partial y} + k\, N1_{y=0}\right] \tag{9}$$

Using the transformed variables (5), above equation reduces to

$$C_f \operatorname{Re}_x^{1/2} = 2\xi^{-1/2}[1+(1-n)K] f''(\xi,0), \text{ where } \operatorname{Re}_x(x) = \sqrt{\frac{u_w x}{\nu}} \tag{10}$$

To solve the system of equations (6) and (7) together with (8), at (τ = 0) ξ = 0, for the initial flow are

Early Unsteady Flow: For 0 < n < 1, ξ = 0, the equations (6) and (7) get reduced to

$$(1+K) f''' + \frac{\eta}{2} f'' + Kg' = 0, \tag{11}$$

$$\left(1+\frac{K}{2}\right)g'' + \frac{\eta}{2}g' + \frac{1}{2}g = 0, \tag{12}$$

together with boundary conditions

$$f(0)=0,\ f'(0)=0,\ g(0)=-nf''(0)\ \ and\ \ f'(\infty)=1,\ \ g(\infty)=0 \tag{13}$$

II. Steady State Flow: The equations for the steady state flow ($\xi = 1$), reduced to

$$(1+K)\, f''' + f\, f'' - f'^2 - Mf' + Kg' = 0 \tag{14}$$

$$\left(1+\frac{K}{2}\right)g'' + f\, g' - f' g - K(2g + f'') = 0 \tag{15}$$

together with the same boundary conditions (13).

Method of Solution: The system of equations (11), (12) and (13) is solved, being linear system, using the method of principle of superposition by obtaining solutions with three different sets of values for the missing initial conditions, namely f′′(0) and g′(0). Having thus obtained initial profiles, the system of equations (6), (7) and (8) is solved for ξ = Δξ, 2Δξ, 3Δξ 4Δξ, ---------1.0 by a procedure to be discussed in what follows.

The derivatives with respect to ξ are replaced by backward difference approximations. The system (6) and (7) is now solved with the initial values f(0) = 0, f′(0) = 0, f′′(0) = α, g(0) = - nα, g′(0) = β. To satisfy the end conditions namely, f′(∞, α, β) -1 = 0 and g(∞, α, β) = 0, we used the Newton's method to find α and β. The derivatives with respect to α and β are obtained by solving the equations (6) and (7) after partial differentiation, with respect to α and β. Thus we solve the system

$$(1+K)F''' + (1-\xi)\frac{\eta}{2}F'' + \xi(fF'' + Ff'' - 2f'F' + KG' = \xi(1-\xi)\frac{\partial F'}{\partial \xi} + M\xi F' \tag{16}$$

$$(1+\frac{K}{2})G'' + (1-\xi)(\frac{G}{2} + \frac{\eta}{2}G') + \xi(fG' + Fg' - f'G - F'g) - K\xi(2G + F'') = \xi(1-\xi)\frac{\partial G}{\partial \xi} \tag{17}$$

$$Here,\ F = \frac{\partial f}{\partial \alpha, \beta}\ and\ G = \frac{\partial g}{\partial \alpha, \beta}$$

This pair of equations is solved once with the initial conditions F(0)=F'(0)=0, F''(0)=1, G(0)=-n, G' (0)=0 and another time with F(0)= F'(0) = F''(0) = 0, G(0) = 0 and G'(0) = 1. The Newton-Raphson method converged in 2 or 3 iterations for each ξ. The accuracy of numerical method is tested with different choices for $\eta_{\max}$, $\Delta\eta$ and $\Delta\xi$. We finally have chosen $\eta_{\max}$=5, $\Delta\eta$ =0.01 and $\Delta\xi$ =0.1.

Each initial value problem is solved using Adam-Bashforth, Adam-Moulton predictor corrector method of fourth order. It may be noted that f, f', f'', g and g', occurring in the equations (16) and (17) are known by solving the equations (6) and (7) for a given α and β. The convergence of solutions is checked on f''(0,ξ) with relative error $<10^{-4}$.

RESULTS AND DISCUSSIONS

The profiles for velocity(f'), microrotation (g) and skin friction coefficient C_f have been drawn for several values of electromagnetic field parameter (M) and material parameter(K), for both early unsteady (o<ξ<1) and steady(ξ=1) cases. To validate our computational code and results, it is compared with the cases of Newtonian flow (K=0, $f_w = 0$) of available results for the steady state case (ξ=1) of Katagiri [5] and Lok Yian et. al.[8] without electromagnetic field parameter (M=0) as in table1. They are well agreed with others. The velocity, microrotation and skin friction profiles of unsteady and steady flow case for various parameter values (K and M) are presented in figures. 2,3, 4 & 5. Both f′and g profiles develop rapidly from rest as ξ increases towards steady state ξ=1. The profiles show a smooth transition from early unsteady flow to the steady flow.

It is also observed that the steep shoot of flow between unsteady and steady cases near the wall. As the electromagnetic parameter (M) increases, the microrotation reduces drastically along the normal direction. In the absence of electromagnetic field parameter, the mcirorotaion is very nominal and not much of a change takes place. The microrotation is more and gets reduced drastically near the wall compared to away from the wall as M reduces which can be seen in the figure 7. This nature is reversed in the case of increase of material parameter (K). The difference in the nature of microrotation profile for n=0 and n=0.5 is observed in figures 6 & 7 and microrotation reduces near the wall with the increase of electromagnetic field parameter (M).

The nature of velocity profiles remain similar to that of material paramer K, during transient flow. On the other hand, the nature of velocity profiles gets changed 0 < **ξ** < 1. It is also noticed that with the increase of material parameter, the skin friction f'' (0, ξ) decreases and has opposite effect as compared to its variation against electromagnetic parameter as can be seen in figure 9. Further the skin friction decreases as the flow approaches the steady state. The results help in controlling the flow to some extent to the desired cases. Increase of material parameter enhances the flow near the wall.

Table 1: Values of the Skin Friction Coefficient $C_f Re_x^{1/2}$ for Values of M=0, K=0, n=0 at ξ=1

Katagiri[2]	LokYian et al.[3]	Present
1.232588	1.232627	1.236651

CONCLUSIONS

It is found that an increase in the material parameter K will increase the velocity of the flow near the wall due to decrease of friction f''(0), where as the effect is reverse with the increase of electromagnetic parameter (M).

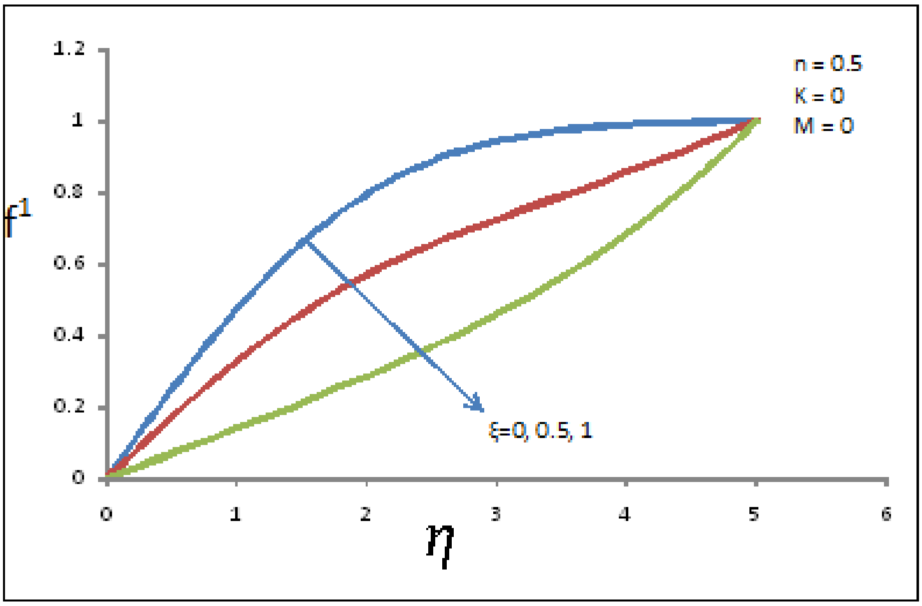

Figure 2: The Variation of Velocity Profiles along Vertical Direction towards Steady State

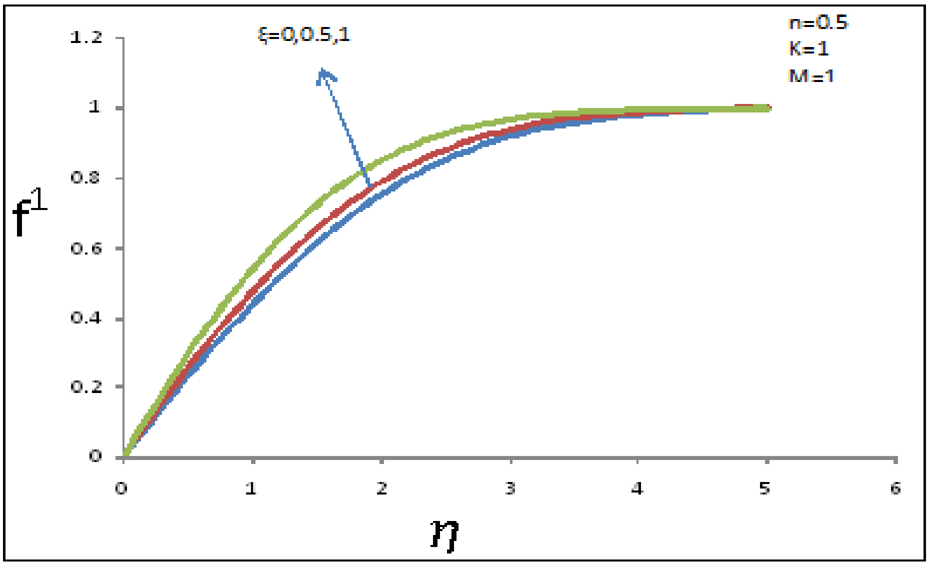

Figure 3: The Variation of Velocity Profiles along Vertical Direction towards Steady State

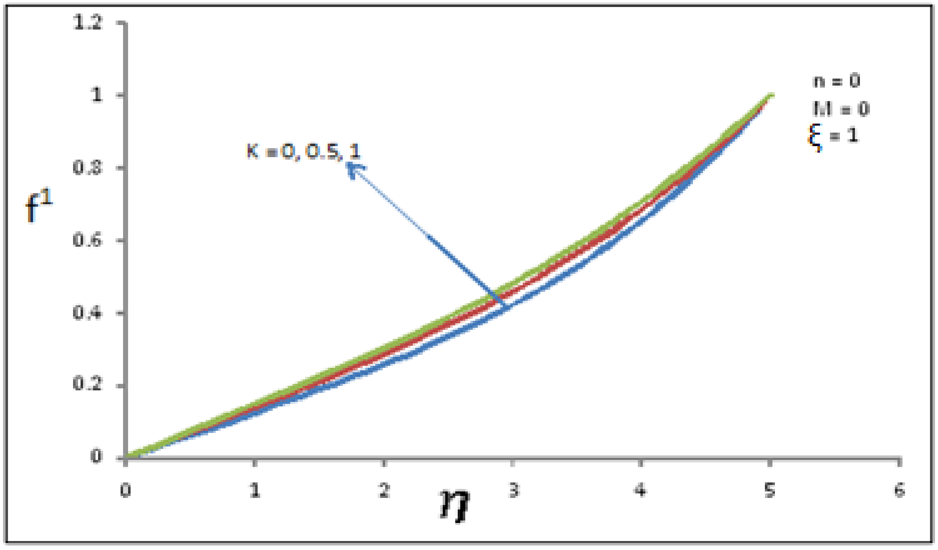

Figure 4: The Variation of Velocity Profiles along Vertical Direction towards Steady State for Various Values of Material Parameter

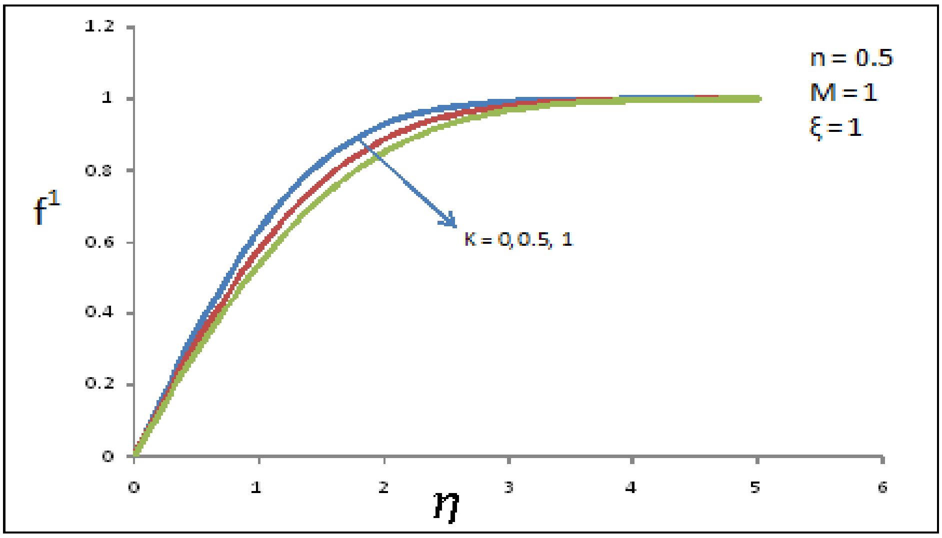

Figure 5: The Variation of Velocity Profiles along Vertical Direction towards Steady State for Various Material Parameter (K) Values

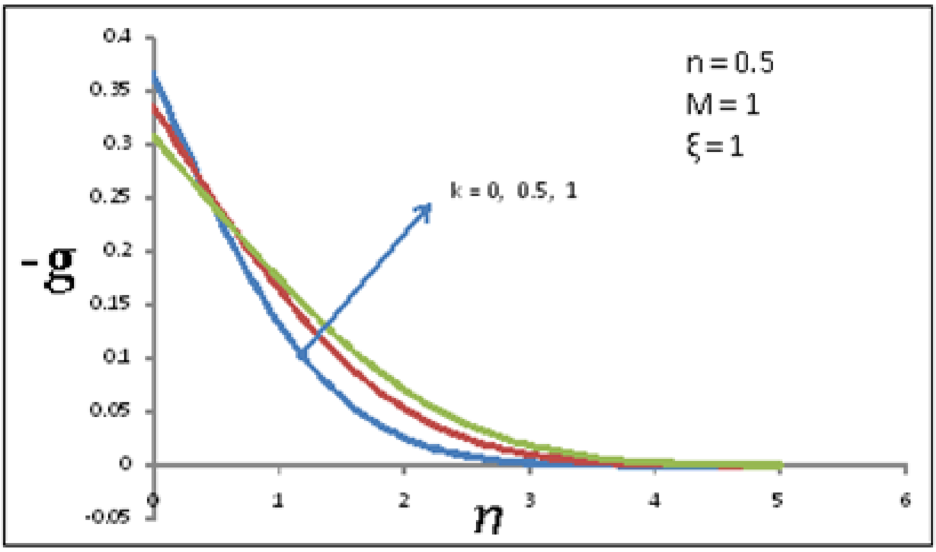

Figure 6: The Micro-Rotation Distribution of Final Steady State Flow

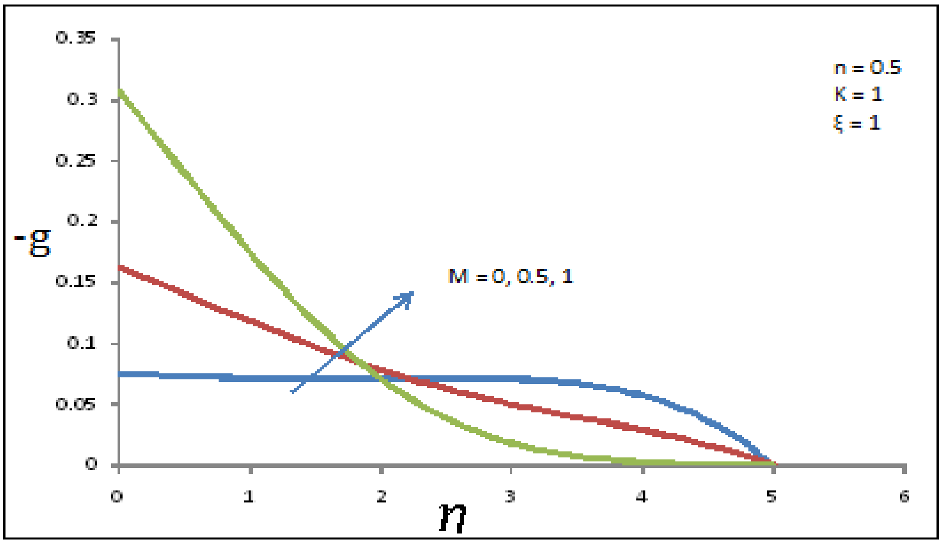

Figure 7: The Variation of Velocity Profiles along Vertical Drection towards Steady State

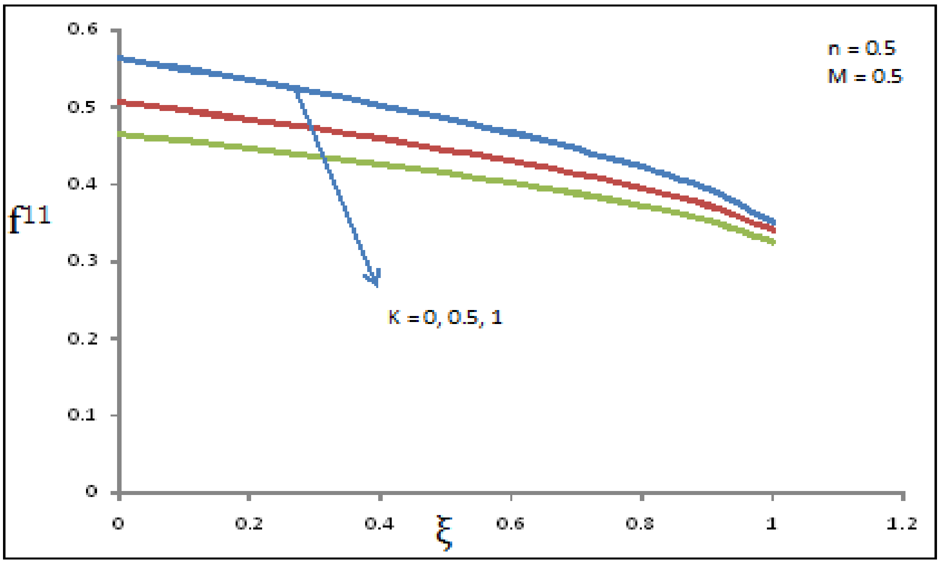

Figure 8: The Variation of Skin Friction f^{11} (0) towards Steady State for Different Values Material Parameter

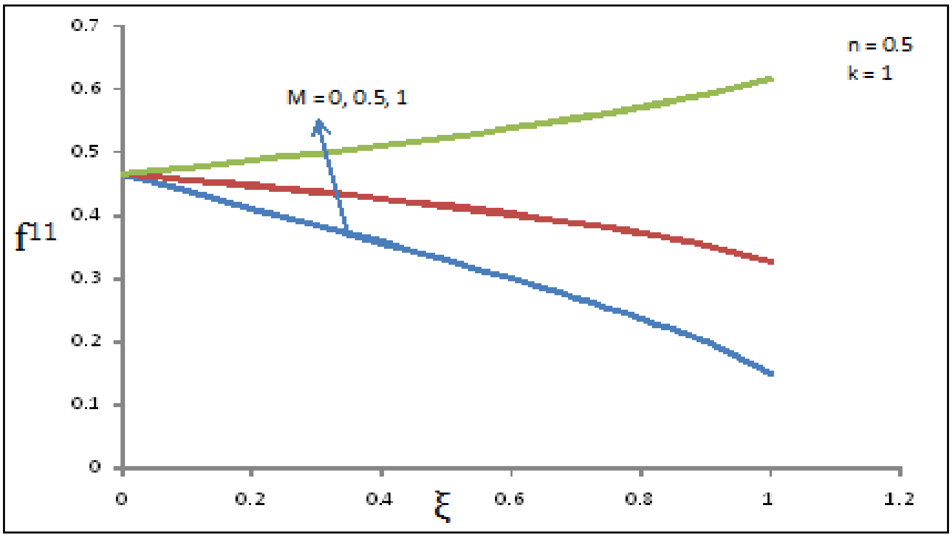

Figure 9: The Variation of Skin Friction towards Steady State for Different Values of Electro-Magnetic Field Parameter

ACKNOWLEDGEMENTS

The authors wish to thank Professor S. R. Koneru for suggesting the basic ideas of computational method used and his encouragement.

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